CONTACT PROBLEMS FOR THE ELASTIC LAYER OF SMALL THICKNESS

(KONTAKTNYE ZADACHI DLIA UPRUGOGO SLOIA MALOI TOLSHCHINY)

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We will investigate the problem of a punch acting on an elastic strip of thickness h : (a) situated, without friction, on a rigid foundation, (b) rigidly attached to a nondeformable foundation. In both cases, we assume that there are no friction forces between the punch and the strip, that the length of the contact line is $2a$, and that the magnitude of $\lambda = h/a$ is small.

The zero term of the asymptotic solution of the above mentioned problems. for small λ , was obtained in [1] in the form of simple and convenient equations. We will refer to an asymtotic of this type as asymmetric. A method is presented below for the determination of the zero term of a simmetric asymptotic, which reflects more accurately the character of the solution, in the form of simpler equations.

It is known, that, using the methods of operational calculus, the problems considered can be reduced to the solution of the integral equation of the form

$$\int_{-a}^{a} q(\xi) K\left(\frac{x-\xi}{h}\right) d\xi = \pi\Delta\gamma(x), \qquad |x| \leqslant a \qquad \left(\Delta = \frac{E}{2(1-\sigma^2)}\right) \tag{1}$$

where $q(\xi)$ is the contact pressure, $\gamma(x)$ is the settlement of the strip boundary in the contact region.

Only the following properties of the kernel $K(k)$ of Equation (1) will be required:

1) the kernel is an even function of k

2) $K(k) \to \pi A\delta(k)$ for $k \to \infty$ $\qquad (k = (x-\xi)/h)$ (2)

Here, $\delta(k)$ is Dirac's delta function for problems (a) and (b) respectively

$$A = 1/2, \qquad A = 1/2\,(1-2\sigma)/(1-\sigma)^2 \tag{3}$$

We will transform Equation (1) to new variables according to

$$x = \frac{h}{\lambda}\sqrt{1-2\lambda t}, \qquad \xi = \frac{h}{\lambda}\sqrt{1-2\lambda\tau} \tag{4}$$

Neglecting in the resulting equation terms of order λ , assuming $1/\lambda = \infty$, and considering the above mentioned properties of the kernel $K(k)$, we get

$$\int_0^\infty q^*(\tau) K(t-\tau)\, d\tau = \frac{\pi \Delta\gamma^*(t)}{h} \qquad (0 \leqslant t < \infty) \tag{5}$$

Thus we obtained the integral equation of the problem for cases (a) and (b).

The approximate solution of these problems, for the condition

$$\gamma(x) = \gamma^*(t) \equiv \gamma$$

is obtained in [1] by the method of Wiener-Hopf, and has the form

$$q^*(t) = \frac{\Delta\gamma}{Ah}\left[\operatorname{erf}\left(\frac{t}{A}\right)^{1/2} + \left(\frac{A}{\pi t}\right)^{1/2} \exp\left(-\frac{t}{A}\right)\right] \qquad (\gamma = \text{const}) \tag{6}$$

Transforming Equation (6) to the variable x according to

$$t = \frac{a^2 - x^2}{2ah} \tag{7}$$

we get the zero term of the symmetric asymptotic of the solution for problems (a) and (b) with small values of the parameter λ and for the case $\gamma(x) \equiv \gamma$ in the form

$$q(x) = \frac{\Delta\gamma}{Ah}\left[\operatorname{erf}\left(\frac{a^2 - x^2}{2Aah}\right)^{1/2} + \frac{1}{\sqrt{\pi}}\left(\frac{2Aah}{a^2 - x^2}\right)^{1/2} \exp\left(-\frac{a^2 - x^2}{2Aah}\right)\right] \tag{8}$$

Approximate solutions of problems (a) and (b), with small λ, for any function $\gamma(x)$, can be obtained from Krein's formula [2]. We will find the magnitude of the force P, acting on the punch, using some of the formulas of the monograph [3]

$$P = \int_{-a}^{a} q(x)\, dx = \pi \Delta\gamma \varkappa \tag{9}$$

$$\varkappa = 2\sqrt{2d/\pi}\, e^{-d} \{2d\, [I_0(d) + I_0(d)] + I_1(d)\} \qquad (d \doteq {}^1/_4 A\lambda) \tag{10}$$

where $I_0(d)$ and $I_1(d)$ are Weber's functions.

The results of calculating $\varkappa$ according to Equation (10) for problems (a) and (b) are given in the Table.

	a)			b) $\sigma = 0.3$		$\sigma = 0.3$	
λ	1	0.5	0.25	1	0.5	1	0.5
$\varkappa$	1.63	2.89	5.42	1.92	3.46	2.01	3.65
$\varkappa$ [1]	1.59	2.86	5.41	1.88	3.44	1.97	3.63
$\varkappa$ [4,5]	1.50	2.82	5.40	1.76	3.42	1.86	3.61

The last two columns of the Table refer to the problem of the interaction of a rigid band with an infinite elastic cylinder. For the problem, Equations (8) to (10) can be obtained in an analogous fashion. However, here

$$A = 1/2\,(1 + \sigma)$$

For comparison, the values of $\varkappa$ calculated from corresponding formulas of [1, 4 and 5], are given in the Table.

It can be seen, that the formulas derived above can be used with confidence for

$$\lambda \leqslant 0.5$$

In conclusion, we note that in a similar fashion we can obtain the zero term of the symmetric asymptotic of the solution for axially symmetric contact problems of an elastic layer of small thickness.

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